AP Calculus AB & BC

Course Overview

The main objective of the course is to give students a thorough understanding of the topics listed in the AB & BC Calculus topics outline. This in turn will give them skills necessary to do well on the AP test and in subsequent college mathematics courses.

Course Outline

Functions

- The year begins with a quick review of functions emphasizing Trigonometric Functions. Other topics include quadratic inequalities, function notation, work with $x+\Delta x$, and the rational root theorem.

Parent Function Matching Activity

A Preview of Calculus

- An Introduction to Limits (Discovery Activity)
- Finding Limits Graphically and Numerically
- Algebraic techniques for evaluating limits
- Techniques for Evaluating Limits Analytically
- Continuity and One-Sided Limits
- Intermediate Value Theorem
- Infinite Limits

Discovery Activity on limits

Differentiation

- Zooming-in activity and local linearity (Discovery Activity)
- Defining Average and Instantaneous Rates of Change at a Point
- Understanding of the derivative – graphically, numerically, and analytically
- Appropriate use of notation – $f'(x)$, y', dy/dx, $f''(x)$, d^2y/dx^2
- Approximating rates of change from graphs and tables of data
- The derivative as: the limit of the average rate of change, an instantaneous rate of change, limit of the difference quotient, and the slope of a curve at a point
- The meaning of the derivative---translating verbal descriptions into equations and vice versa
- The relationship between differentiability and continuity
- The Derivative and the Tangent Line Problem

- Basic Differentiation and Rates of Change
- Product and Quotient Rules
- Higher Order Derivatives
- The Chain Rule
- Implicit Differentiation
- Related Rates

Discovery Activity – Local Linearity
Peanut M&M Activity
Student Presentations of related rate problems

Applications of Differentiation

- Extrema on an Interval
- Rolle's Theorem and the Mean Value Theorem
- Increasing and Decreasing Functions and the First Derivative Test
- Concavity and the Second Derivative Test
- Relating the graphs of f, f', and f''
- Limits at Infinity
- Tangent line to a curve and linear approximations
- A summary of Curve Sketching
- Optimization Problems
- Application problems including position, velocity, acceleration, and rectilinear motion
- Parametric Equations (BC only)

F, F' Matching Activity
F, F', F" Around the World Game

Integration

- Antiderivatives and indefinite integration, including antiderivatives following directly from derivatives of basic functions
  Basic properties of the definite integral
- Area under a curve
- Meaning of the definite integral
- Definite integral as a limit of Riemann sums
- Riemann sums, including left, right, and midpoint sums
- Trapezoidal sums
- Use of Riemann sums and trapezoidal sums to approximate definite integrals of functions that are represented analytically, graphically, and by tables of data
- Connecting Riemann sums to sigma notation

- Use of the First Fundamental Theorem to evaluate definite integrals
- Use of substitution of variables to evaluate definite integrals
- Integration by substitution, using long division and completing the square
- Integrating using Integration by Parts (BC only)
- Integration using Partial Fractions (BC only)
- Evaluating Improper Integrals (BC only)
- The Second Fundamental Theorem of Calculus and functions defined by integrals
- The Mean Value Theorem for Integrals and the average value of a function
- Understanding the difference between an indefinite and a definite integral

Logarithmic, Exponential, and other Transcendental Functions

- The Natural Logarithmic Function: Differentiation
- The Natural Logarithmic Function: Integration
- Inverse Functions: Differentiation and Integration
- Bases other than e and Applications
- Inverse Trigonometric Functions: Differentiation
- Inverse Trigonometric Functions: Integration
- L'Hopital's Rule

Differential Equations

- Solving separable differential equations
- Applications of differential equations in modeling, including exponential growth
- Use of slope fields to interpret a differential equation geometrically
- Drawing slope fields and solution curves for differential equations
- Approximating solutions using Euler's Method (BC only)
- Logistic models with differential equations (BC only)

Slope Field Activity

Applications of Integration

- The integral as an accumulator of rates of change
- Area of a region between two curves
- Volume of a solid with known cross sections
- Volume of solids of revolution

- Arc Length in rectangular and parametric forms (BC only)
- Applications of integration in problems involving a particle moving along a line, including the use of the definite integral with an initial condition and using the definite integral to find the distance traveled by a particle along a line
- Area and Arc Length in Polar Coordinates (BC only)

How Sweet it is activity
Volumes of Solids Matching Activity
Volumes with Known Cross Sections Project

Integration Techniques
- Integration by Parts
- Improper Integrals
- Trigonometric Substitution
- Partial Fractions
- Logistic Differential Equations
- L'Hopital's Rule

Infinite Series (BC ONLY)

- Convergence and divergence of sequences
- Definition of a series as a sequence of partial sums
- Convergence of a series defined in terms of the limit of the sequence of partial sums of a series
- Introduction to convergence and divergence of a series with technology
- Geometric series and applications
- The *n*th-Term Test for Divergence
- The Integral Test
- The Integral Test
- Comparisons of series
- Alternating series and the Alternating Series Remainder
- The Ratio and Root Tests
- Taylor polynomials and approximations: introduction using the graphing calculator
- Power series and radius and interval of convergence
- Taylor and Maclaurin series for a given function
- Maclaurin series for sin x, cos x, e, e^x, and $\frac{1}{1-x}$
- Manipulation of series, including substitution, addition of series, multiplication of series by a constant and/or a variable, differentiation of series, integration of series, and forming a new series from a known series

- Taylor's Theorem with the Lagrange Form of the Remainder (Lagrange Error Bound)

Discovery Activity on convergence and divergence

Teaching Strategies

Problem Solving

Topics are approached using the Rule of Four. We look at concepts numerically, graphically, analytically, and verbally. Throughout the year students work together on a regular basis and are encouraged to form study groups. Students are often asked for verbal explanations, both oral and written, to give them the opportunity to communicate their work and their reasoning in words. They are asked to discuss the meaning of their answers, particularly when working related rates problems, optimization problems, and applications of integration problems. I also ask them to discuss the steps they have used to arrive at their answers. Activities are included to emphasize discovery learning whenever possible.

Justification of Answers.

I ask my students to justify their answers on homework, quizzes, and tests, and I prefer that they write the justifications in sentences. We talk a lot about the amount of work they need to show and about the correct way to justify their work on various types of problems.

Technology

Most students own their own graphing calculators but we have a class set of TI-84+ calculators. The calculator is used to support discovery as well as conclusions.

Examples:

1) When introducing limits, the students use their graphing Calculator to make a table of values to help them determine the limit of $f(x)=\dfrac{\sin(3x)}{x}$ as x gets closer and closer to zero.

2) When the students are introduced to the Squeeze Theorem, I have them graph $y=x,\ y=-x,$ and $y=x\cos\left(\dfrac{50\pi}{x}\right)$ in radian mode on the same screen over the x-interval from -1 to 1, and then they zoom in on the graph at x = 0 to see how the Squeeze Theorem can be used to find $\lim\limits_{x\to 0} x\cos\left(\dfrac{50\pi}{x}\right)$.

3) We discover the meaning of local linearity and differentiability by using the zooming in feature of the calculator to zoom in on the graphs of several functions and to determine which ones will look linear if we zoom enough times and which ones will not.

4) When I introduce derivatives and local linearity, I have the students zoom in on a particular graph at a given point until their graph looks linear. Then they estimate the slope at the given point by using a second point very close to the original point. One group of students chooses a second point with a smaller x coordinate then the original point, and another group chooses a second point with a larger x-coordinate. Then we discuss how we could estimate the actual value of the derivative at the given point by using our results and how the number of times the students zoomed in effects the accuracy of their estimate.

5) The students discover the relationship between the graph of a function and the graph of its derivative by graphing several functions and their derivatives on their calculators and then discussing what happens on the graph of the derivative for the points where the function is increasing and decreasing and where the function is concave up and concave down.

6) When we study Riemann sums and how they can be used to estimate the area bounded by a curve and the x-axis on a given interval, we use a calculator program to see how increasing the number of subintervals effects the accuracy of our estimate.

Many homework problems and about half of the problems on quizzes and tests are done without the use of the graphing calculator. Since the AP Exam is half calculator and half non-calculator, I feel that it is very important for students to have practice working problems both ways. We spend time in class discussions talking about the types of questions that they must know how to work *with* their calculators and the types of questions that they must know how to work *without* their calculators. We also discuss the techniques needed to use the calculator most efficiently (storing functions in the y = screen, storing values that will be used later in the problem, etc.).

Student Presentations

From the very beginning of the first semester, in addition to textbook exercises, free response style problems are handed out as new concepts are introduced and students are encouraged to work on them as the unit unfolds. Emphasis is always placed on justification and explanations incorporating proper vocabulary and mathematical notation. Students work in collaborative groups to encourage communication about the math.

The goal is always to finish with the topics leaving 3 weeks for review before the AP test.

Student Evaluation

Homework is assigned through out each unit and is due on quiz and test days. Short quizzes are given often and are a combination of multiple-choice and open-ended. Two to three times per week, short 10 question quizzes are administered to check for memorization of facts and formulas.

Primary Textbook

Larson and Edwards, Calculus of a Single Variable. 10^{th} Edition. Boston, MA: Cengage Learning, 2017, 2014.

Additional Resources

AP Central

Student Activities

1. Parent Function Matching Activity- Student pairs are given the graphs of 20 parent functions from lines to trigonometric functions and are expected to match them to the correct equation without a calculator. The activity is designed to encourage students to discuss the domain and range of each function.

2. Peanut M&M Activity - This is an activity I learned at an AP conference. It is done just before the Chain Rule is taught and emphasizes to the student to look for the "inside" of the function. Students will relate the peanut to the notation used for derivatives of composite functions. They will be introduced to the notation f '(g(x))(g'(x))

3. F, F’ Matching Activity - This activity consists of 4 sets of cards. There is a set of function graphs, verbal descriptions of the graphs, graphs of the derivatives, and verbal descriptions of the derivatives. Students work in groups to form books of 4 cards related to the same function. The activity is designed to encourage communication between students as they compare and contrast the different functions and their derivatives.

4. F, F’, F” Around the World Game- Each desk has a set of cards relating to the First Derivative Test, Test for Concavity, and the Second Derivative Test. Students are given the beginning of a phrase and must complete it with the correct card. Students who answer incorrectly remain seated while those who answer correctly move to the next available seat. The first student back to their original seat wins.

5. Slope Field Activity-After an introduction to slope fields is presented students match equations to the given slope fields, draw slope fields, and solve differential equations using slope fields. This activity is done in small groups to allow students to collaborate.

6. Volumes of Solids Matching Activity – This activity consists of 3 sets of cards. There is a set of, verbal descriptions, the integral set-up, and the solution. Students work in groups to form books of 3 cards related to the same solid of revolution. The activity is designed to encourage communication between students as they compare and contrast the different functions and their integrals.

7. Local Linearity - We discover the meaning of local linearity and differentiability by using the zooming in feature of the calculator. We start with y = sin x and restrict the window to $-1x10^{-5}$ to $1x10^{-5}$ for both the x and y values. Students attempt to write the equation of the function based on what they see. Then the window is changed to reveal the sine function. Students then use the zoom function to zoom in on the graphs of several functions and to determine which ones will look linear if we zoom enough times and which ones will not.

8. Students use their calculator to investigate the meaning of a limit by looking at graphs and filling out tables as a function approaches a value.

9. Students use their calculator to investigate the meaning of convergence and divergence of both sequences and series.

Student Projects

Related Rates Project – Students create their own related rate problem and illustrate it and include all necessary information. They trade in class and work each other's problems.

Volumes with Known Cross Sections Project-Students design, make, and solve a solid with known cross-sections.